The Broadcast Approach in Communications Systems: Overview, Applications and Perspectives

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Outline

- Overview of the broadcast approach
- Broadcasting related results
  - Partial transmit channel state information (CSI), diversity-multiplexing-tradeoff (DMT), multiple-input multiple-output (MIMO) channel, hybrid automatic repeat request (ARQ).
  - **Figures of merit**: Expected throughput, expected delay, expected distortion.
  - **Cooperating network nodes**: Two hop relay, two-colocated receivers, multi-session cooperation.
- Research Outlook and Perspectives

The Case for a Broadcast Approach

Consider a state dependent channel (Cover '72)

$$P(Y|X,S)$$

Y - channel output $Y \in \mathbb{Y}$.
X - channel input $X \in \mathbb{X}$.
S - channel state parameter, where $S \in \mathbb{S}$.

- This is viewed as a broadcast setting where each possible state S is associated with a different receiver.
- Given a probability space for $\mathbb{S}$, problems of average type (rate/delay/distortion) performance are motivated.

★ Reliable transmission with rate **adapted** to actual channel realization, **without** a-priori channel knowledge (Cover'72), (Equitz-Cover'91), (Rimoldi'94), (Shamai'97).

The Block Fading Channel Model

$$\mathbf{y} = h\mathbf{x} + \mathbf{n} \tag{1}$$

$\mathbf{x}_{1\times N}$ - transmitted vector, with power constraint $\frac{1}{N}E[\mathbf{x}\mathbf{x}^\dagger] \leq P$.

$\mathbf{y}_{1\times N}$ - received vector.

$\mathbf{n}_{1\times N}$ - additive white Gaussian noise (AWGN), with iid elements $CN(0,1)$.

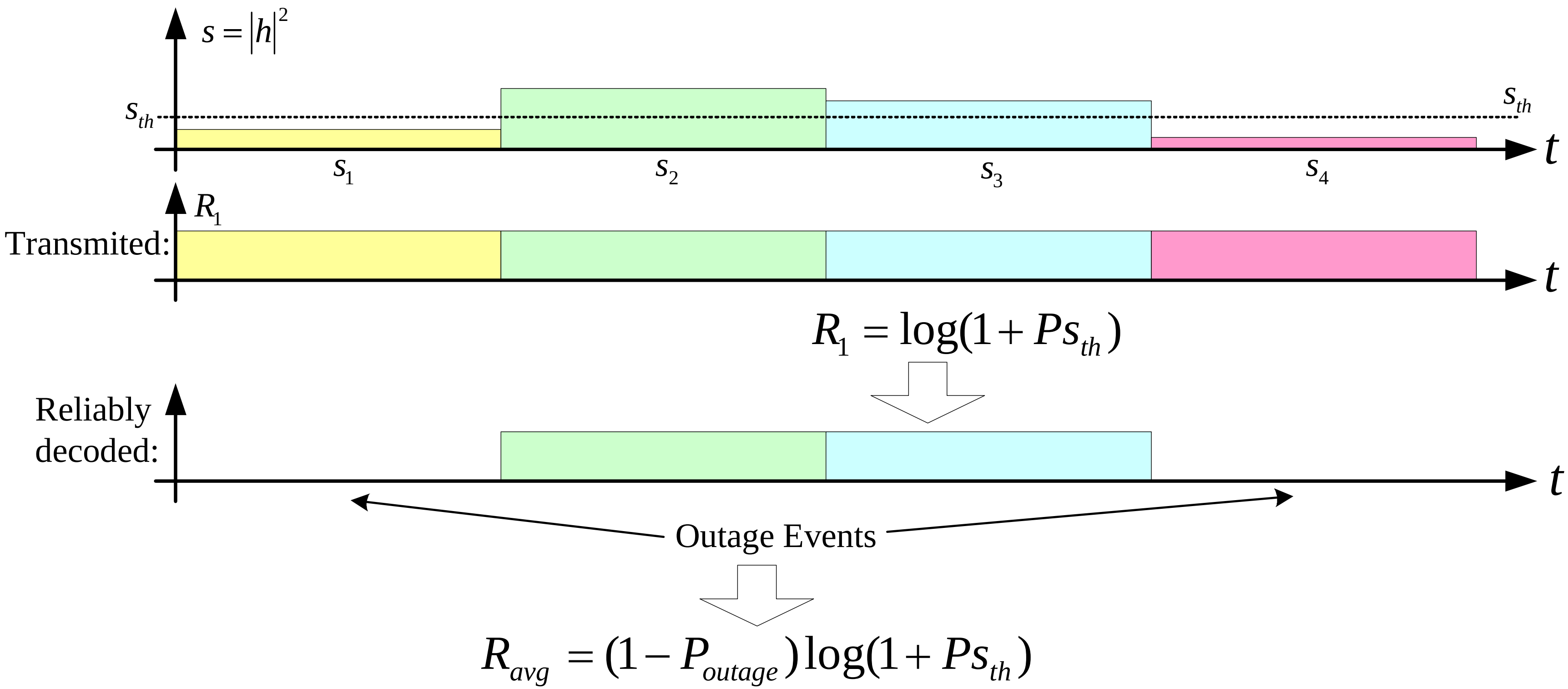
h - fading coefficient, perfectly known by the receiver, fixed over every block, $CN(0,1)$ iid distributed over multiple blocks.

$s = |h|^2$ - channel state parameter.

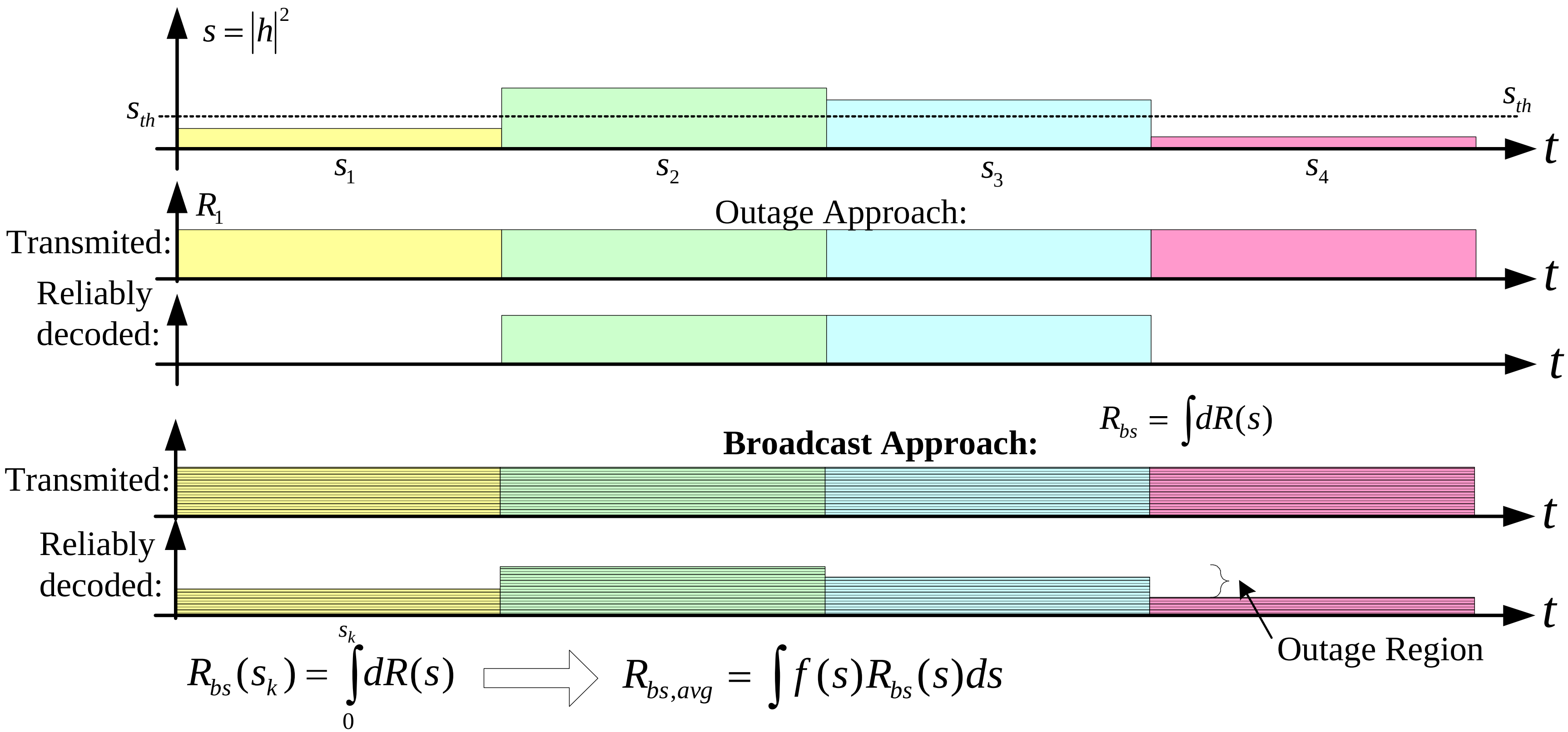
h remains fixed during every transmission block.

Details in (Shamai '97) (Shamai-Steiner-IT '03).

Outage Approach



Broadcast vs. Outage



Broadcast Approach - Definitions

- s - fading gain designating SNR of a virtual receiver.
- Transmitter views a *degraded* broadcast channel.
- $R(s)$ - the reliably conveyed information rate at fading level s.
- Power assigned to the s-th stream $\rho(s)ds$.
- Information streams indexed by $u > s$ are undetectable $\Longrightarrow$ residual interference $I(s)$.

Broadcast Approach - Overview

- Incremental differential rate

$$dR(s) \; = \; \log\left(1 + \frac{s\rho(s)ds}{1 + sI(s)}\right) \; = \; \frac{s\rho(s)ds}{1 + sI(s)} \tag{2}$$

- The residual interference power is

$$I(s) = \int_{s}^{\infty} \rho(u)du \tag{3}$$

- Total transmitted power P is

$$P = \int_{0}^{\infty} \rho(u)du = I(0) \tag{4}$$

Broadcast Approach - Overview (cont.)

- Reliable rate at fading level s

$$R(s) = \int_0^s \frac{u\rho(u)du}{1+uI(u)} \quad (5)$$

- Expected achievable broadcasting rate

$$R_T = \int_0^\infty du\; f(u)R(u) = \int_0^\infty du(1-F(u))\frac{u\rho(u)}{1+uI(u)} \quad (6)$$

$f(u)$ - the pdf of the fading power, $F(u) = \int\limits_0^u da f(a)$ - the corresponding cdf.

- Optimization problem: maximize expected rate over the input power distribution

$$R_{T,max} = \max_{I(u)} \int_0^\infty du(1-F(u))\frac{u\rho(u)}{1+uI(u)} \equiv \max_{I(u)} \int_0^\infty duS(u,I,I_u) \quad (7)$$

where $I_u \equiv \frac{dI(u)}{du} = -\rho(u)$.

Broadcast Approach - Overview (cont.)

Proposition 1 *The power distribution, which maximizes the expected throughput in (7) is*

$$I(x) = \begin{cases} \frac{1-F(x)-x\cdot f(x)}{x^2 f(x)} & , \; x_0 \leq x \leq x_1 \\ 0 & , \; else \end{cases} \tag{8}$$

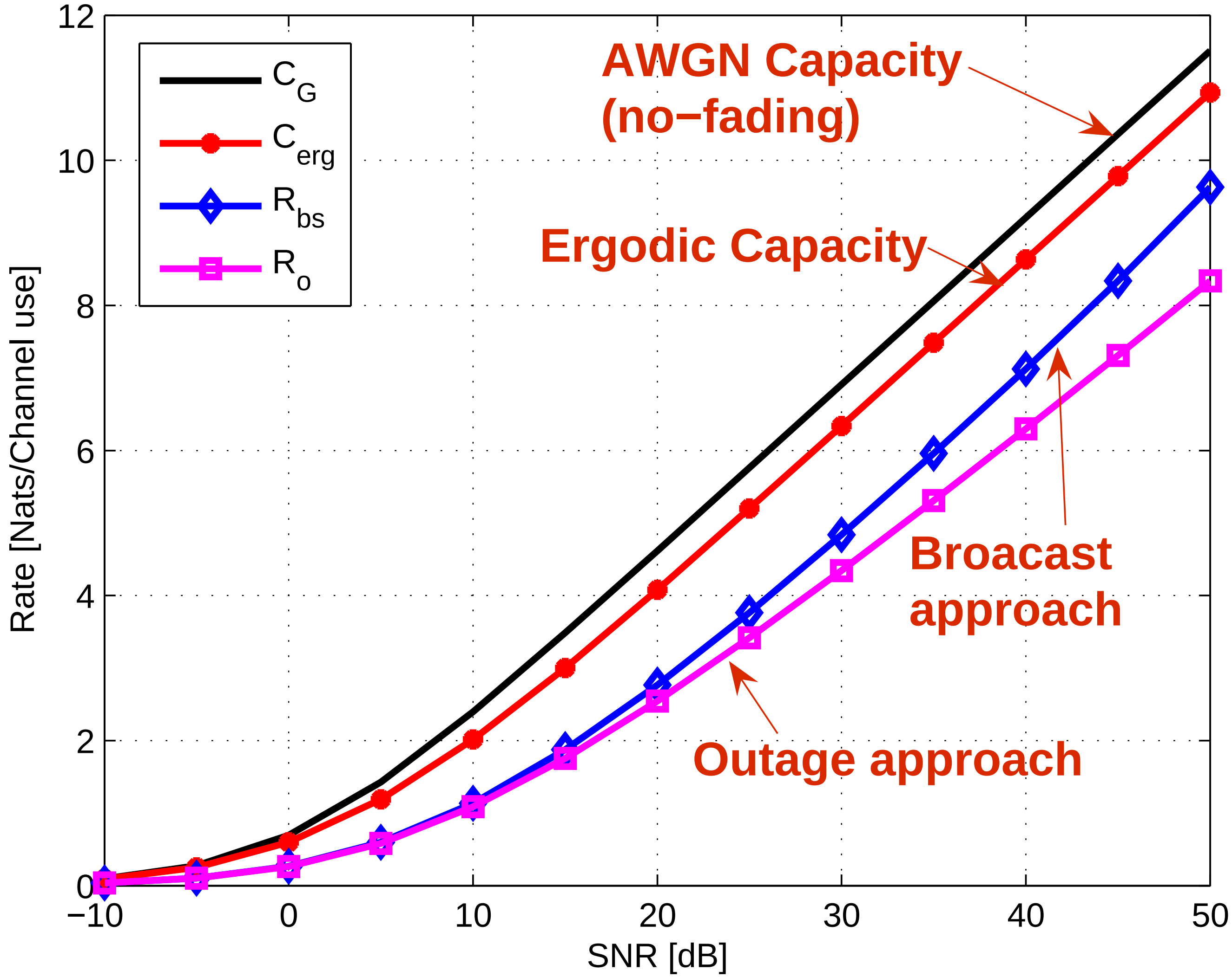
where x_0 is determined by $I(x_0) = P$, and x_1 by $I(x_1) = 0$.

Proof: The extremum condition for the functional (7) is given by the associated Eüler equation

$$S_I - \frac{d}{du} S_{I_u} = 0$$

which simplifies from a differential equation into a linear equation by $I(u)$, yielding (8).

The Rayleigh Fading Case



SISO broadcasting - Summary

- Reviewed continuous broadcasting upper bound derivation for expected throughput maximization.
- Results extend straightforwardly to multiple-input single-output (MISO) and single-input multiple-output (SIMO) channels.
- Pronounced gains were demonstrated over a Rayleigh fading channel.
- Broadcasting gains compared to the outage approach may significantly change for a different figure of merit (e.g. expected delay, expected distortion, etc.).

Partial Transmit CSI

- Only a partial CSI description is available at the transmitter.
- We study broadcasting w.r.t. the CSI uncertainty distribution.
- Partial transmit CSI (TCSI) models
  - Quantized feedback.
  - Stochastic Gaussian model.
- Related work (sample)
  - Quantized feedback as partial TCSI (Moustakas-Simon'03).
  - Finite level coding with quantized feedback (Kim-Skoglund '05).
  - Stochastic Gaussian TCSI model (Visotsky-Madhow '01),(Moustakas-Simon '03), (Xie-Georghiades-Arapostathis '05).

Details in (Steiner-Shamai-TWC March'08).

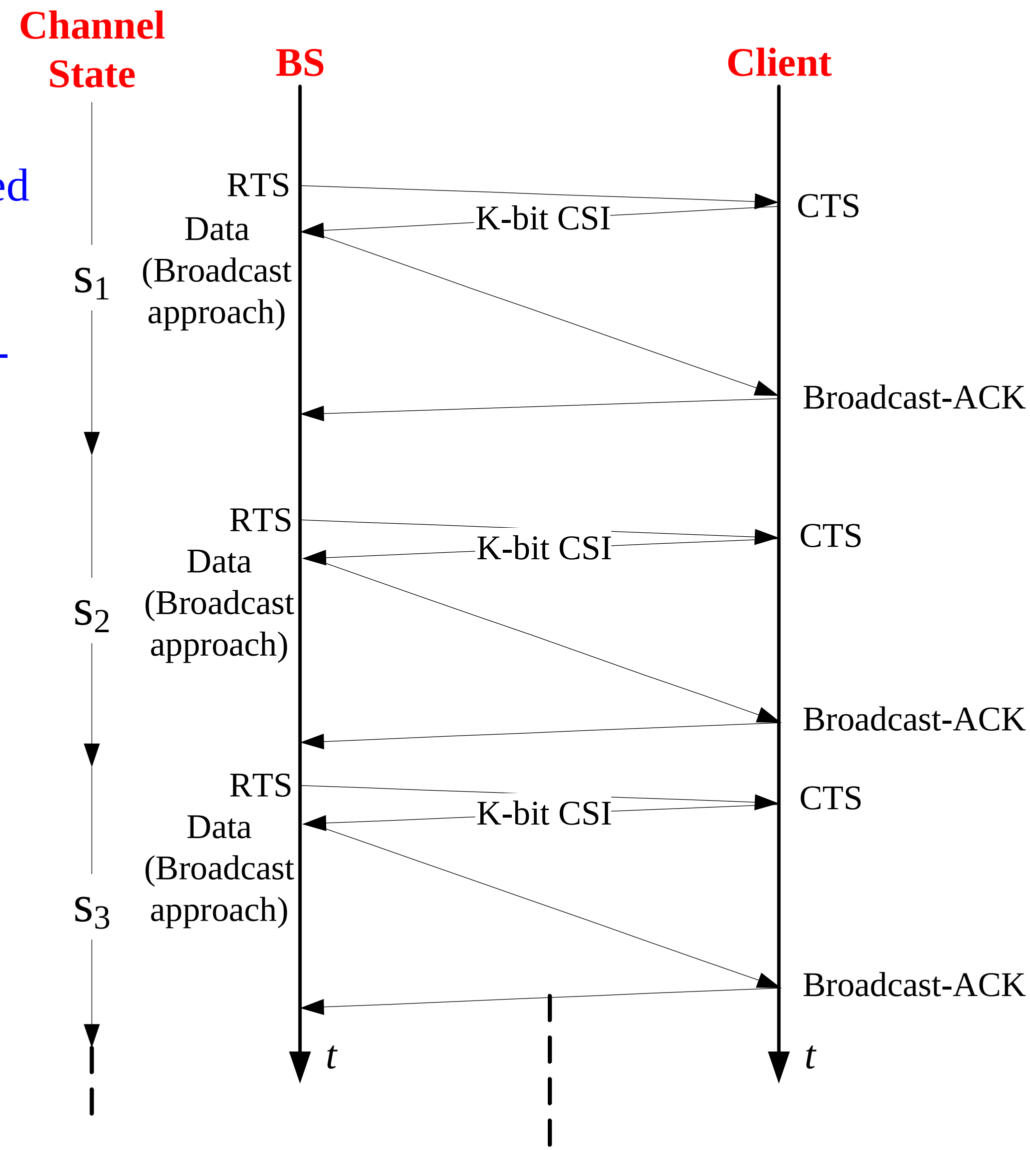
Quantized Feedback - Example

Handshake protocol example

1. Every data transmission is initiated with RTS/CTS.

2. Every CTS includes additional K-bits of feedback.

3. The protocols overhead (RTS/CTS) is negligible relative to the downlink payload.



Quantized Feedback

- A CSI feedback of $N = \lceil \log_2(K) \rceil$ bits specifies intervals

$$\{\mathcal{I}_k\}_{k=1}^{K} = \{[s_{th,0}, s_{th,1}), [s_{th,1}, s_{th,2}), ..., [s_{th,K-1}, s_{th,K})\} \quad (9)$$

- A broadcast approach is applied for every interval $\mathcal{I}_k$.
- The general expression for the expected rate

$$R_{avg} = \sum_{k=1}^{K} P_\nu(\nu \in \mathcal{I}_k) \cdot R_{avg,k} \quad (10)$$

- $P_\nu(\nu \in \mathcal{I}_k)$ - probability of fading interval $\mathcal{I}_k$.
- $R_{avg,k}$ - expected achievable rate (outage/broadcasting) for $\mathcal{I}_k$.

Single-Bit Feedback

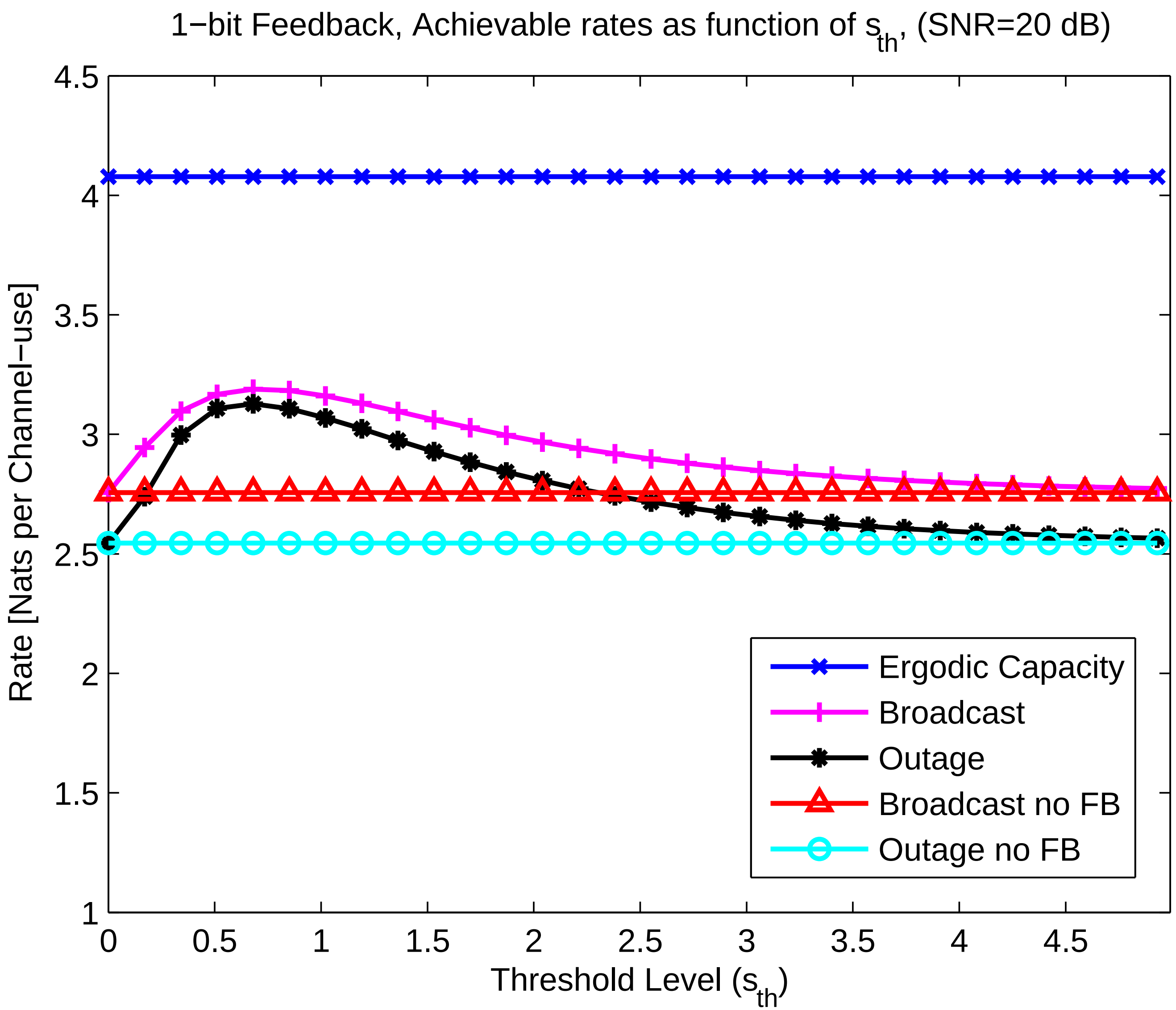
Proposition 2 *Maximal achievable rates, in presence of a single bit feedback, are given by (Rayleigh fading)*

$$R_{avg,1} = (1-e^{-s_{th}})^{-1}(2E_1(s_0^{(1)}) - 2E_1(s_1^{(1)}) + e^{-s_1^{(1)}} - e^{-s_0^{(1)}} + e^{-s_{th}}(s_1^{(1)} - s_0^{(1)} - 2\log\left(\frac{s_1^{(1)}}{s_0^{(1)}}\right))) \quad (11)$$

$$R_{avg,2} = \begin{cases} e^{s_{th}}(2E_1(s_0^{(2)}) - 2E_1(s_1^{(2)}) - (e^{-s_0^{(2)}} - e^{-s_1^{(2)}})) & s_{th} < s_0^{(2)} \\ 2e^{s_{th}}(E_1(s_{th}) - E_1(s_1^{(2)}) - e^{-s_{th}} + e^{-s_1^{(2)}}) + \log[(1+s_{th}P)s_{th}] & s_{th} \in [s_0^{(2)}, s_1^{(2)} \\ \log(1+s_{th}P) & s_{th} > s_1^{(2)} \end{cases}$$

where $s_0^{(k)}$, *and* $s_1^{(k)}$ *are determined by* $I_k^{opt}(s_0^{(k)}) = P$, *and* $I_k^{opt}(s_1^{(k)}) = 0,\ k = 1, 2$.

Quantized Feedback - Numerical Results



Partial TCSI - Summary

- Optimal power allocation and maximal achievable rates are derived for quantized feedback.
- For a single-bit feedback the broadcasting gain (over outage) is roughly $\sim$1 dB (for Rayleigh fading).
- With higher resolution feedback the broadcasting gain nearly vanishes (for Rayleigh fading).
- Under a stochastic Gaussian partial TCSI model:
  - SISO broadcasting achievable rates were obtained.
- **Rapidly approximating full-CSI conditions (Rayleigh fading) $\Rightarrow$ outage approach is optimal**.
- **Open problems**
  - Long term power constraints (Outage: time-domain waterfilling).
  - Extensions to SIMO (straightforward), MIMO.

Diversity-Multiplexing Tradeoff (Overview)

- Define a multiplexing gain r and a diversity order d as

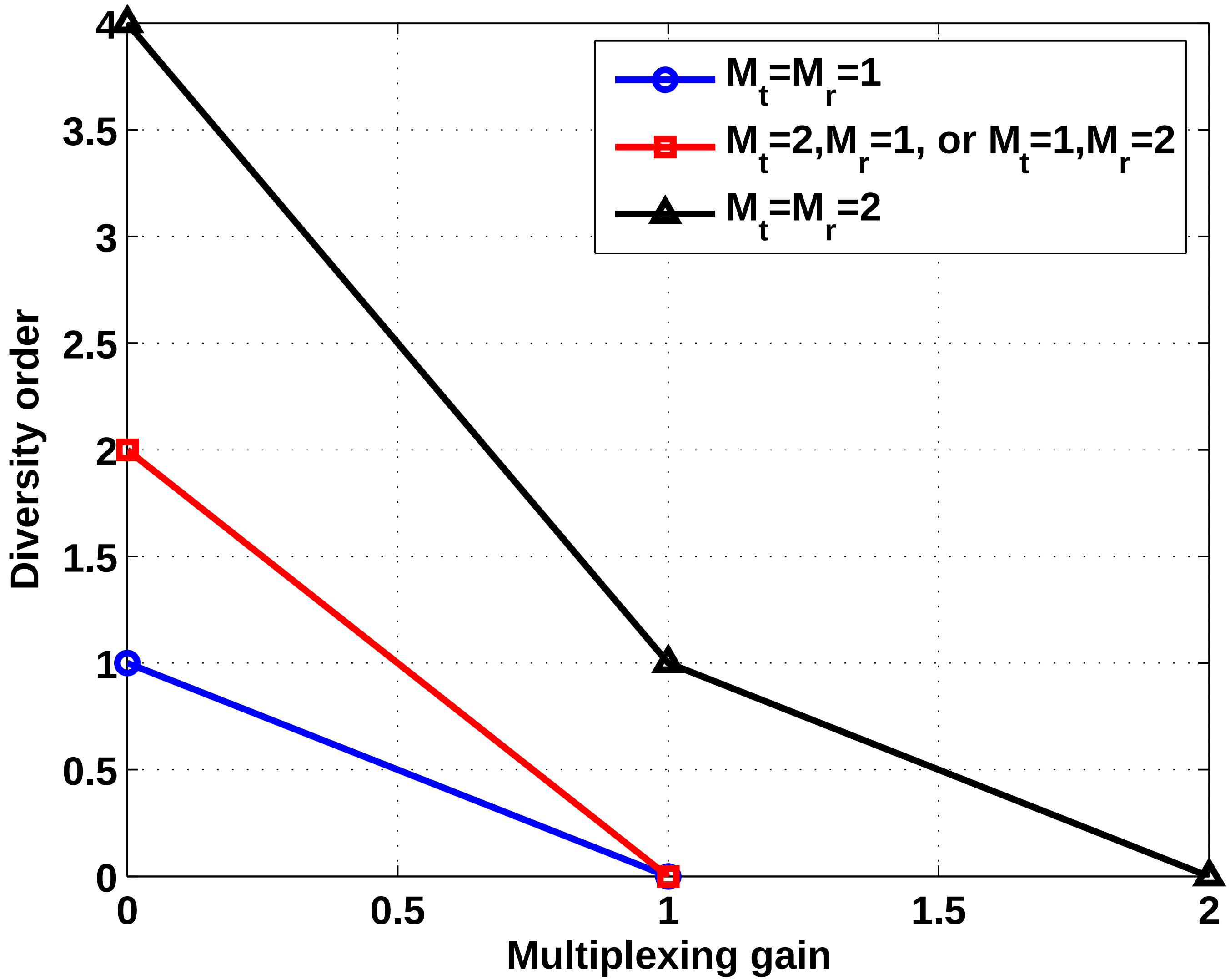
$$\lim_{\mathsf{SNR}\to\infty} \frac{R(\mathsf{SNR})}{\log(\mathsf{SNR})} = r \ , \qquad \lim_{\mathsf{SNR}\to\infty} \frac{\log P_e(\mathsf{SNR})}{\log(\mathsf{SNR})} = -d$$

- **Theorem 1** *(Zheng-Tse '02) For $T > M_t + M_r - 1$, and $K = \min(M_t, M_r)$, the optimal trade-off curve $d^*(r)$ is the piece-wise linear function connecting $(k, d^*(k))$, $k = 0, ..., K$,*

$$d^*(k) = (M_t - k)(M_r - k)$$

T - block length; M_t - number of transmit antennas; M_r - number of receive antennas.

Diversity-Multiplexing Tradeoff - Example



Two Level Coding DMT

- Power allocated to each layer: $(\mathsf{SNR}_1, \mathsf{SNR}_2)$, s.t. $\mathsf{SNR}_1 + \mathsf{SNR}_2 \leq P_s$, and $\mathsf{SNR}_1/\mathsf{SNR}_2 = \mathsf{SNR}^{\beta}$.
- Limit to ONE degree of freedom, i.e. $\min(M_t, M_r) = 1$.
- Denote the diversity order of k^{th} layer by $\widetilde{d}_k$.
- **Theorem 2** *(Diggavi-Tse '04) When* $\min(M_t, M_r) = 1$*, the characterization of the rate tuple* $(r_1, \widetilde{d}_1, r_2, \widetilde{d}_2)$ *is*

$$\begin{aligned} \widetilde{d}_1 &< d^*(r_1) \\ \widetilde{d}_2 &< d^*(r_2) \\ \min(\widetilde{d}_1, \widetilde{d}_2) &< d^*(r_1 + r_2) \end{aligned}$$

Continuous Broadcasting DMT

- Is the DMT of a single degree-of-freedom (DoF) channel infinitely divisible?
- **Theorem 3** *(Diggavi-Tse '04) Let* $\min(M_t, M_r) = 1$*, given* L *layers, with multiplexing gains* $r_1, ..., r_L$*, and outage diversity orders* $\widetilde{d}_1 < \widetilde{d}_2 < ... < \widetilde{d}_L$*, respectively, then the rate region is described by*

$$\widetilde{d}_k < d^* \left(\sum_{n=1}^{k} r_n \right), \qquad k = 1, ..., L$$

$\Rightarrow$ Using continuous broadcasting, for a single DoF channel, the DMT can **be achieved at every point simultaneously**.

More Broadcasting DMT Related Results

- On successive refinement of diversity (Digavi-Tse '04).
- Design of practical layered space-time coding schemes (Diggavi-Calderbank-Dusad-Al-Dhahir '08).
- Layering for fading inter-symbol interference (ISI) MISO channels (Dusad-Diggavi '08).
- DMT characterization with multi-layer coding and degraded message sets for the parallel channel (Diggavi-Tse '06).
  For $\min(M_t, M_r) > 1$, the DMT curve cannot be attained simultaneously (Digavi-Tse '06).
  Optimal simultaneously achievable DMT?
- and more...

The MIMO Broadcast Approach

- In the broadcast approach: layering is equivalent to transmitting over a broadcast channel to multiple users.

$\Longrightarrow$ Transmission over a non-degraded MIMO broadcast channel with many receivers.

- Channel ranking via majorization is considered.
- All possible realizations of H are visualized as channels referring to a continuum of ranked users.
- For simplicity we discuss the case of $\min\{M, N\} = 2$.
- Transmitted signal x is composed of a layered double indexed data stream with indices u and v associated with the eigenvalues of the matrix $\frac{1}{M}HH^{\dagger}$.

Details in (Shamai-Steiner-IT '03).

MIMO Broadcasting - Related Work

- Optimal rate allocation for block fading channels (Liu-Lau-Takeshita-Fitz '02).
- The MIMO Broadcast channel capacity (Weingarten-Steinberg-Shamai'06).
- Broadcasting over uncertain channels with decoding delay constraints (Whiting-Yeh '06).
- Approximately universal coding over slowly fading channels (Tavildar-Viswanath '06).

Variational Problem Definition

- Let $u = \lambda_2$, $v = \lambda_1$ $(\lambda_1 \geq \lambda_2)$ for some channel realization.
- The incremental rate $d^2 R(u,v)$

$$d^2 R(u,v) = \frac{u\rho(u,v)\,dudv}{1 + uI(u,v)} + \frac{v\rho(u,v)\,dudv}{1 + vI(u,v)}\,, \qquad (12)$$

  where $\rho(u,v)dudv$ - power associated with the information stream indexed by $(u,v),\ (v \geq u)$.
- By majorization: for the pair (u,v), following layers (a,b), with $a \leq b$, such that

$$0 \leq a \leq u, \quad a + b \leq u + v$$

  can be decoded.

Variational Problem Definition (cont.)

- Residual interference at (u,v).

$$I(u,v) = P - \int_0^u da \int_a^{v+u-a} db\, \rho(a,b).$$

where the power density is $\rho(u,v) = -\frac{\partial^2}{\partial uv} I(u,v) \triangleq I_{uv}$.

- Achievable rate for (u,v)

$$R(u,v) = -\int_0^u da \int_a^{v+u-a} db \left[\frac{aI_{ab}(a,b)}{1+aI(a,b)} + \frac{bI_{ab}(a,b)}{1+bI(a,b)}\right].$$

with expected rate $R_{av} = \int\int dudv\, f(u,v)\, R(u,v)$.

Variational Problem Definition (cont.)

Proposition 3 *The expected rate for the MIMO broadcast approach with* $\min(M, N) = 2$ *is given by*

$$J[I] = \int_0^\infty du \int_0^\infty dv \, (1 + F(u, v) - F(u) - F(v)) \cdot \left[\frac{uI_{uv}(u, v)}{1 + uI(u, v)} + \frac{vI_{uv}(u, v)}{1 + vI(u, v)} \right]$$

where $J[I] \triangleq \int \int du dv S(u, v, I, I_{uv})$.

Proof: The main ingredient here is integration in parts, together with the properties of the eigenvalues joint distribution.

MIMO Broadcasting - Extremum Condition

Proposition 4 *The condition for extremum of the achievable expected throughput $J[I]$ via majorization for the $\min(M, N) = 2$ MIMO broadcast approach*

$$S_I + \frac{\partial^2}{\partial uv} S_{I_{uv}} = 0. \tag{13}$$

Proof: This is a second order partial differential equation (PDE) of the Eüler equation (Gelfand-Fomin'63).

- Define the variation h on I, such that $\Delta J = J[I + h] - J[I]$.
- First order Taylor series approximation for the integrand gives δJ.
- Use Green's theorem for simplifying the PDE.

The second order PDE (13) does not lend itself to a closed form analytical solution...

MIMO broadcasting - Summary

- Channel ranking via majorization was considered.
- An extremum condition in the form of a second order PDE was obtained via variational calculus tools.
- Sub-optimality of majorization was demonstrated through outage capacity.
- Single level coding, P_{out} derived in (Simon-Moustakas '03),

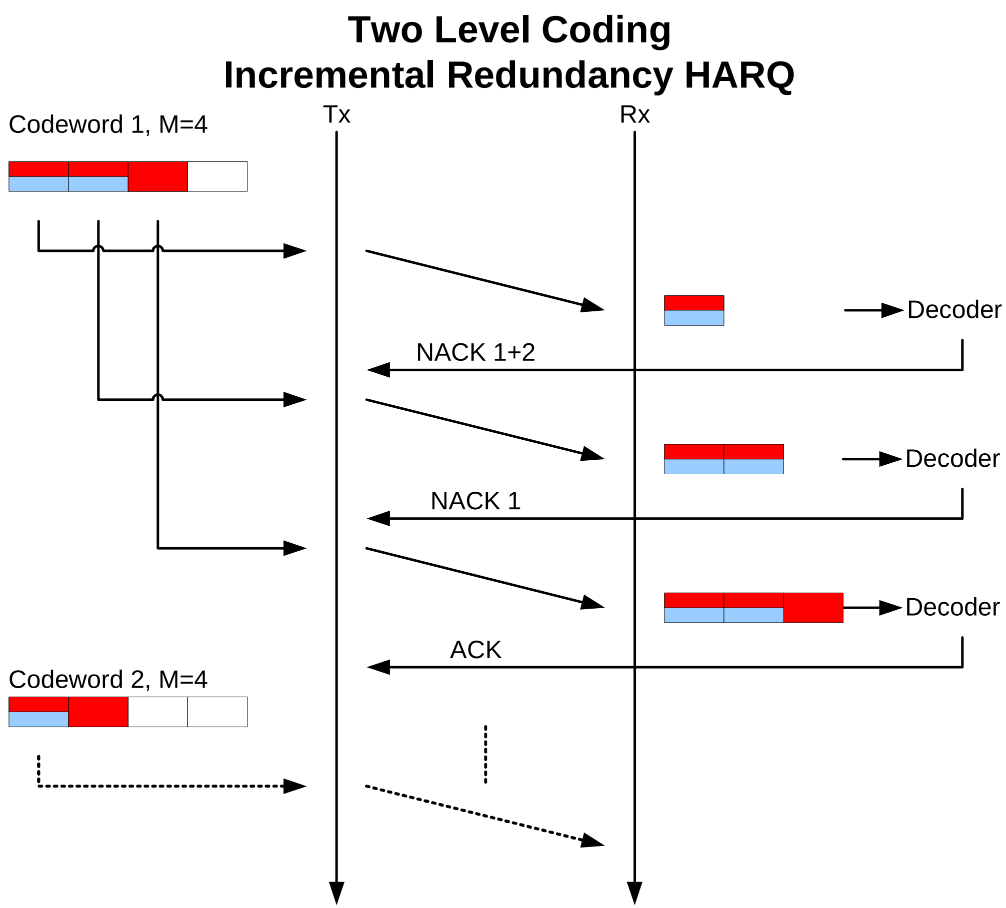
$$C_{out} = \max_{R} (1 - P_{out})R, \quad \text{where} \quad P_{out} = Pr\{\log\det(I + \frac{P}{M} H H^{\dagger}) < R\}$$

- Two level coding
  - Expected throughput single-integral expressions were obtained under: optimal decoding and (sub-optimal) successive decoding rules.
  - Robustness to sub-optimal power allocation under optimal decoding rule (unlike the successive decoding receiver case).
  - Marginal gains over the outage capacity - the 'hardening' effect in MIMO limits the potential gains of a broadcast approach.

MIMO broadcasting - Open Problems

- Optimal power distribution for MIMO broadcasting over fading channels.
- Challenges: No degradation ordering!
- Two state channel with individual rates, the capacity is known (also for more than 2 users) (Weingarten-Steinberg-Shamai ’06).
- The results with common rate are not yet solved in full (Weingarten-Steinberg-Shamai ISIT’06).
- Degraded message set problem is solved in full for two users (Weingarten-Steinberg-Shamai ISIT’06) based on (Korner-Marton ’77), however it does not generalize easily to more than two states (Nair-El Gamal ’08).

Broadcasting IR-HARQ Strategies



Details in (Steiner-Shamai TWC’08, to appear).

HARQ & Broadcasting - Related Work (a sample)

- Chase-Combining (CC) H-ARQ (Chase'85), sub-packet retransmission (Yiqing-Jiangzhou'06).
- Incremental-Redundancy (IR) H-ARQ (Sesia-Caire-Vivier'04), (Kim-Hur-Ramamoorthy-McLaughlin'06), (Soijanin-Varnica-Whiting'06).
- Repetition protocols with partial transmit CSI (Tuninetti'08).
- Layered ARQ with SNR feedback (Erez-Trott-Wornell'08).
- The diversity-multiplexing-delay tradeoff for MIMO IR-HARQ (El Gamal-Caire-Damen'06).
- Collaborative H-ARQ relaying protocols (Tabet-Dusad-Knopp'05), (Stanojev-Simeone-Bar-Ness'06).

Continuous Broadcasting HARQ, Protocol-I

- Transmitter prepares a continuously layered coded packet over $s \in [s_0, s_1]$.
- Only one retransmission is allowed ($M = 2$)
- **Protocol I Broadcast-CC (BCC)/Broadcast-IR (BIR):**
  - $s < s_0$ - Nothing decoded on first transmission, retransmission for all layers.
  - $s \in [s_0, s_1)$, **and** $s_{eq}(s) < s_1$ - Retransmission for the subset of layers which can be reliably decoded.
  - $s \in [s_0, s_1)$, **and** $s_{eq}(s) = s_1$ - Retransmission of undecoded layers and an additional new information single layer.
  - $s \geq s_1$ - new information single layer is transmitted on retransmission.

Protocol-I (cont.)

Proposition 5 *The average achievable throughput of BCC/BIR-HARQ with Protocol-I over a two block quasi-static fading channel is*

$$\eta_{\textit{Protocol-I}} = \frac{1}{2}\Big(R_{bs,(s<s_0)} + R_{bs,(s_0 \le s < s_{eq} < s_1)} + R_{bs,(s_b<s<s_1)} + R_{1L,(s \ge s_1)}\Big) \quad (14)$$

where $E[D] = 2$*, since* $M = 2$*.*

- **Protocol-II:** Identical to protocol-I, with the following exceptions:
  - For $s < s_0$: NO retransmission
  - Expected inter-renewal time $E[D] = 2 - F(s_0)$

Outage Approach Retransmission Protocol

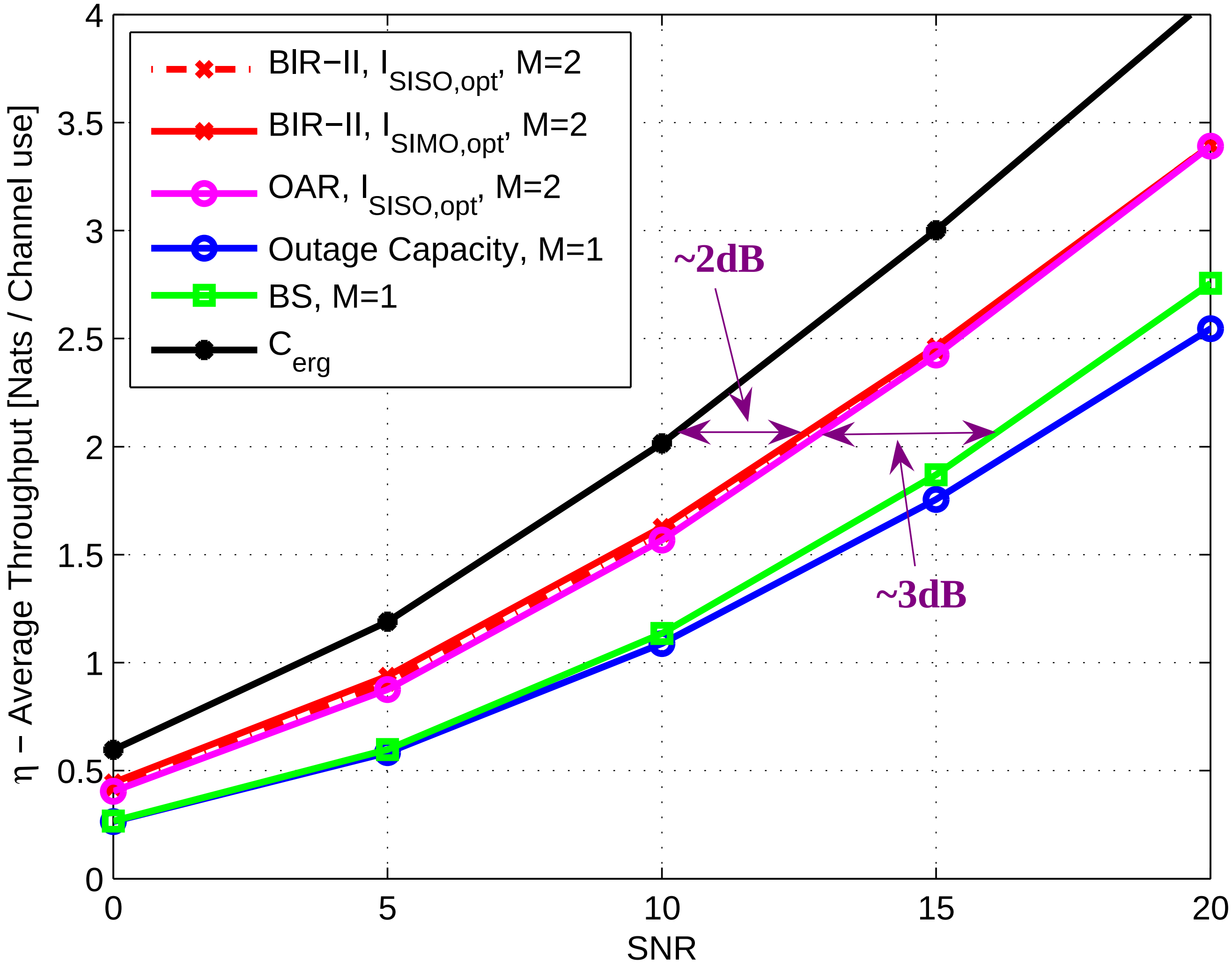
- **Outage Approach Retransmission (OAR) Protocol:**
  - $s < s_0$ - NO retransmission.
  - $s \in [s_0, s_1)$ - Retransmission of a new single layer in a rate matched to s.
  - $s \geq s_1$ - Retransmission of a new single layer in a rate matched to s_1.
- **Proposition 6** *The average throughput of the OAR protocol over a two block quasi-static fading channel is*

$$\eta_{OAR} = \frac{1}{2 - F(s_0)} \left(R_{bs,SISO} + R_{erg,(s_0 \leq s \leq s_1)} + R_{1L,(s \geq s_1)} \right)$$

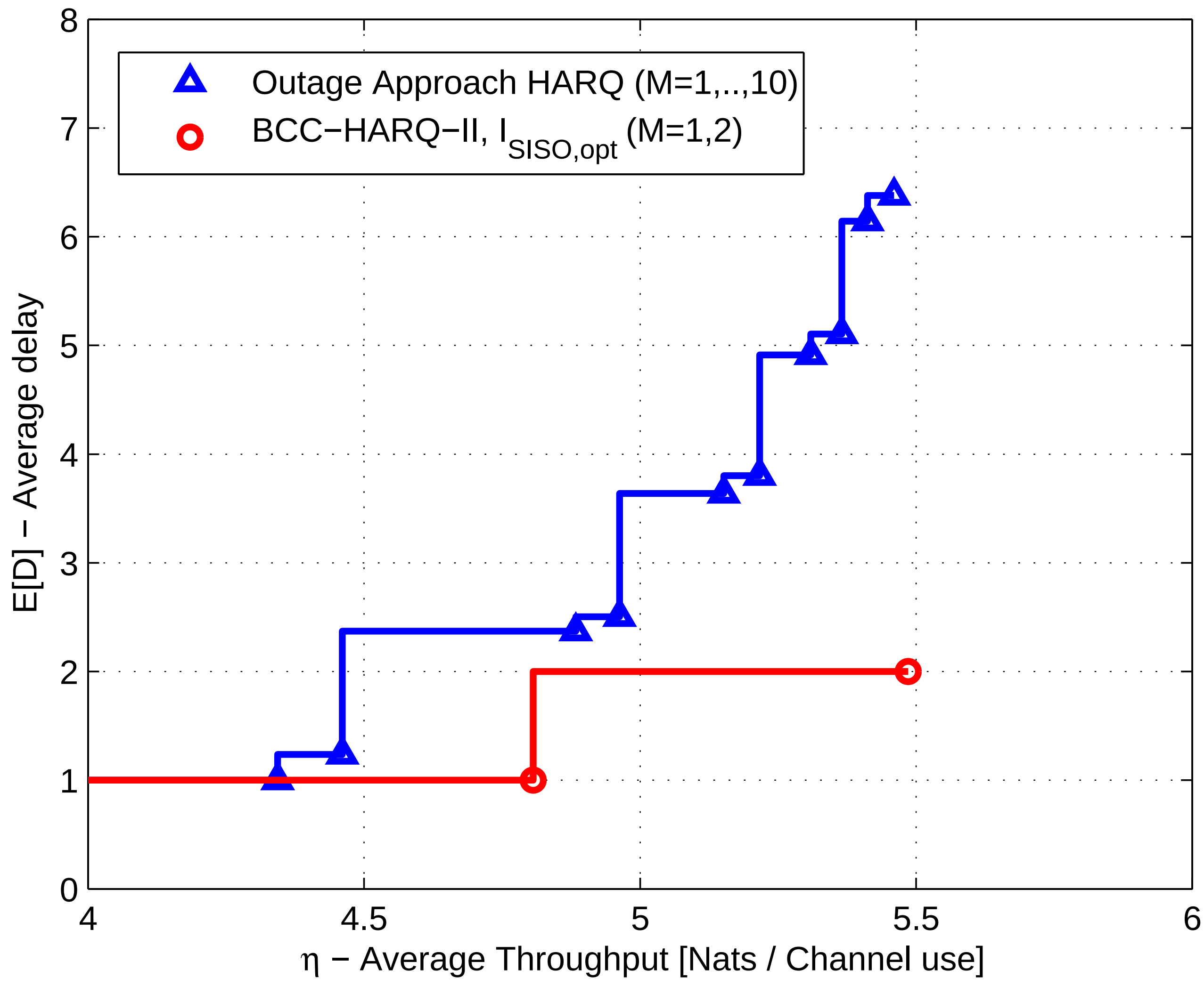
where $E[D] = 2 - F(s_0)$*, and*

$$R_{erg,(s_0 \leq s \leq s_1)} = \int_{s_0}^{s_1} ds f(s) \log(1 + sP)$$

BIR-HARQ Protocol-II and OAR-Protocol



HARQ Expected Delay: Outage vs. Broadcasting



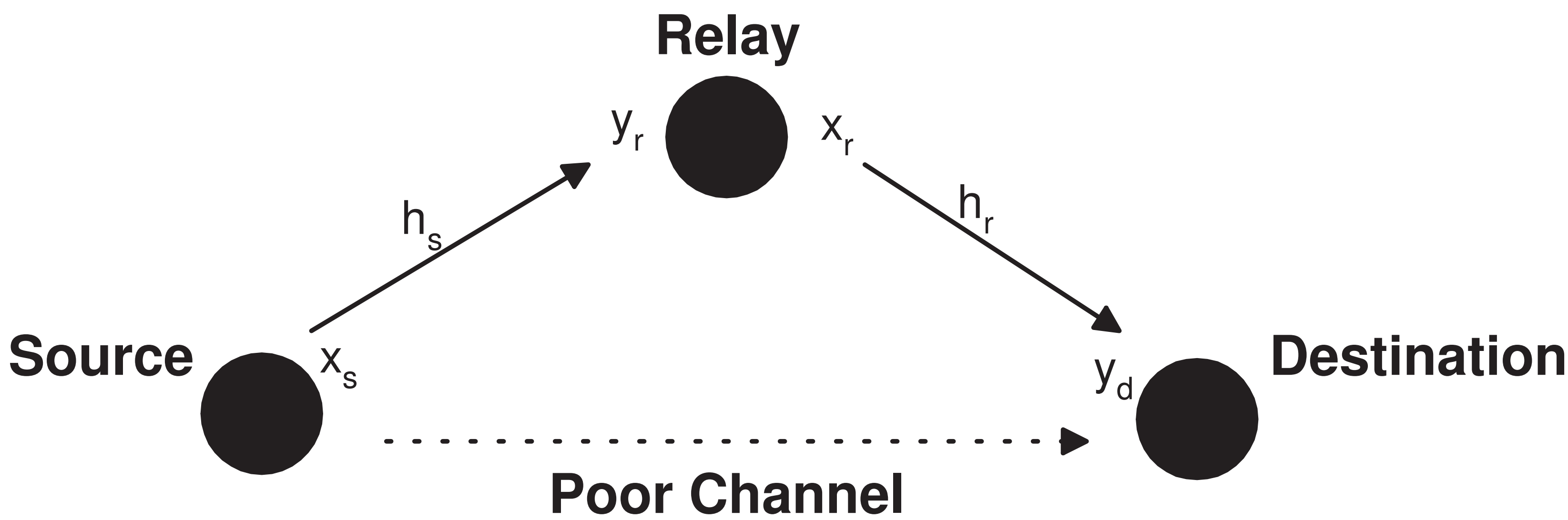
Broadcasting and HARQ - Summary

- Multi-layer broadcasting HARQ Protocols were presented.

$\Rightarrow$ High throughput is achieved with low latency.

- Continuous broadcasting yields pronounced gains over the outage approach even at **low** SNRs.
- OAR Protocol is rather straightforward for implementation and is highly efficient.
- BIR/BCC-HARQ were evaluated for sub-optimal power distributions.
- **Open problems**
  - Optimal power distribution for BIR/BCC HARQ.
  - Extensions to SIMO/MISO (straightforward), MIMO.
  - Variable power allocation on retransmissions.

The Two-Hop Relay Channel



How can the communication over a two-hop fading relay channel benefit from layering?

Details in (Steiner-Shamai-IT ’06).

Broadcasting and Relaying - Related Work

- Practical settings of two-hop relay channels for ad-hoc wireless networks (Zhu-Cao '05), (Hung-Gitlin '04).
- Some information theoretic results for the general relay channel (Cover-El Gamal '79), (Host-Madsen-Zhang '03), (El Gamal -Mohseni-Zahedi '04), (Kramer-Gastpar-Gupta '04). The MIMO relay channel (Wang-Zhang-Host-Madsen '05). Large scale networks (Leveque-Telatar '05), (Gupta-Kumar '03). Cooperative communications tutorial (Kramer-Maric-Yates '07).
- Broadcasting and relaying protocols for two co-located users (Katz-Shamai '05).
- Broadcast strategies with time division protocols for the relay channel (Yuksel-Erkip '04).

Decode-Forward (DF) Protocols

- **Outage approach** - both source and relay perform single level coding.
- **Source: Outage, Relay: Multi-layer Coding**.
- **Source: Layering, Relay: Outage**. The relay transmits in a single rate matched to the successful decoded rate.
- **Naive broadcasting** - the relay performs sub-optimal continuous broadcasting.
- **Optimal broadcasting** at source and relay - intractable optimization problem.

The Amplify-Forward (AF) Relay

Relay input is scaled by $\gamma = \sqrt{\frac{P_r}{P_s|h_s|^2+1}} \longrightarrow$ relay power P_r.
The equivalent received signal at the destination

$$\mathbf{y}'_d = \frac{\gamma h_r h_s}{\sqrt{\gamma^2|h_r|^2+1}}\mathbf{x}_s + \mathbf{n}'_r \ , \tag{15}$$

$\mathbf{n}'_r \sim \mathcal{CN}(0,1)$. Destination views an equivalent fading gain

$$s_b = \frac{\gamma^2 s_r s_s}{\gamma^2 s_r + 1} = \frac{P_r s_r s_s}{P_r s_r + P_s s_s + 1} \ , \tag{16}$$

$s_r = |h_r|^2$, $s_s = |h_s|^2$. For Rayleigh fading, cdf of s_b is directly derived.

Maximal achievable rates are explicitly obtained.

Hybrid Amplify-Quantize-Forward (AQF)

Relay input amplified to P_r, then quantized - with a single codebook.

Equivalent quantized signal is now

$$u_q = \beta\gamma h_s x_s + \beta\gamma n_s + n'_q \tag{17}$$

where $\beta = 1 - \frac{D}{P_r}$, and γ is the AF coefficient.

Outage approach:

$$R_{AQF,1,avg} = \max_{s_s,D} \overline{P}_{out} \cdot \log\left(1 + \frac{s_s P_s}{1 + \frac{D(1+s_s P_s)}{P_r - D}}\right), \tag{18}$$

$$\overline{P}_{out} = \text{Pr}\left(\log\frac{P_r}{D} \leq \log(1 + \nu_r P_r) \bigcap \nu_s \geq s_s\right)$$

Quantize-Forward (QF) versus Hybrid-AQF

- AQF requires only one codebook, whereas QF has a separate codebook for every ν_s.
- **Proposition 7** *The relay-destination throughput as function of* $D(\nu_s)$

$$D^*(\nu_s) = \arg \max_{D(\nu_s)} \int_0^\infty d\nu_s \int_0^\infty d\nu_r e^{-\nu_s - \nu_r} \log \frac{1+\nu_s P_s}{D(\nu_s)} \cdot \mathbf{1}\left(\frac{1+\nu_s P_s}{D(\nu_s)} \leq (1+\nu_r P_r) \right)$$

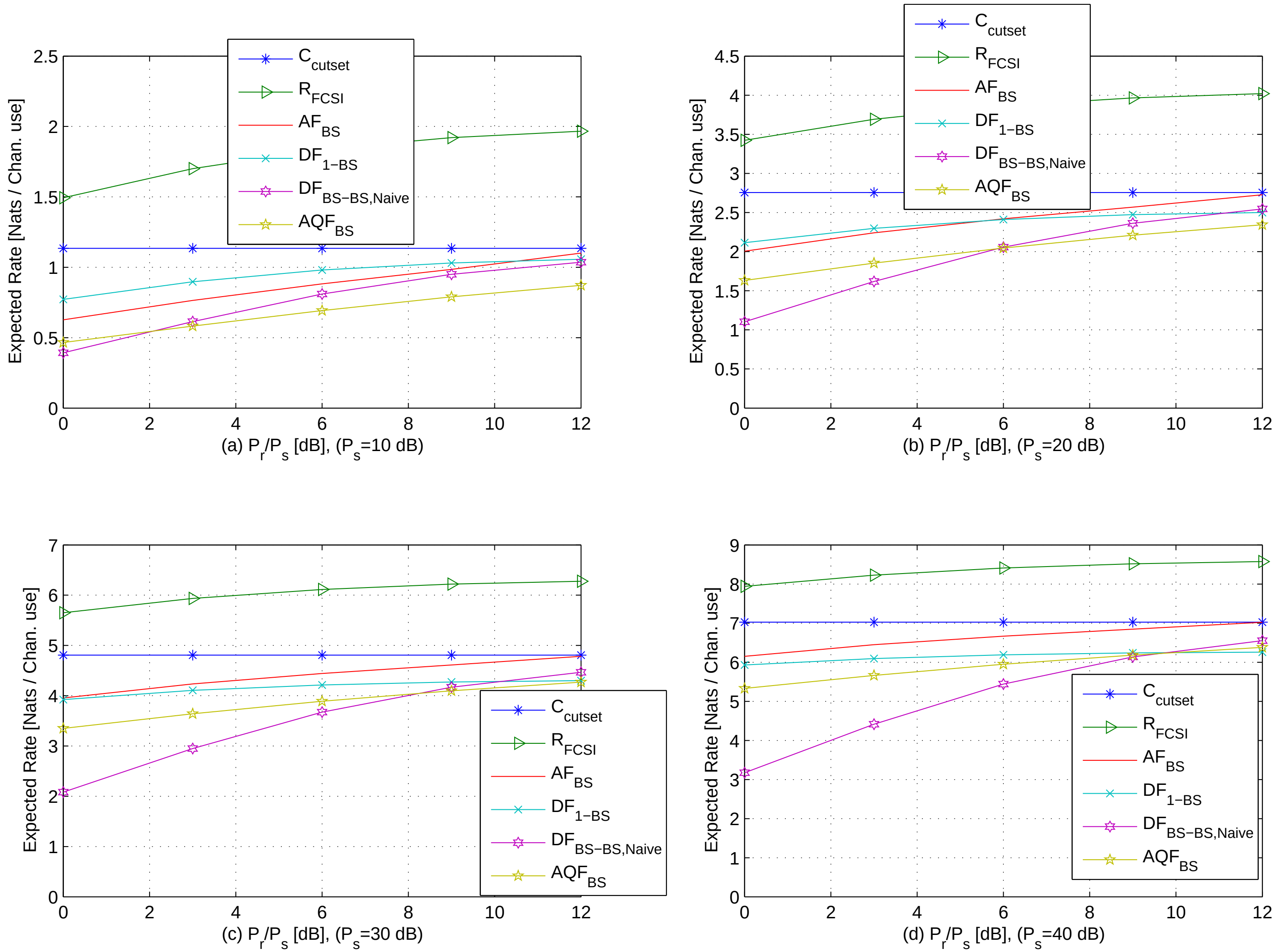
is maximized by

$$D^*(\nu_s) = \alpha(1+\nu_s P_s) \tag{19}$$

$\alpha = \frac{1}{P_r} W_L(P_r)$, $W_L(x)$ - *Lambert W-function.*

AQF is optimal in terms of relay-destination link throughput.

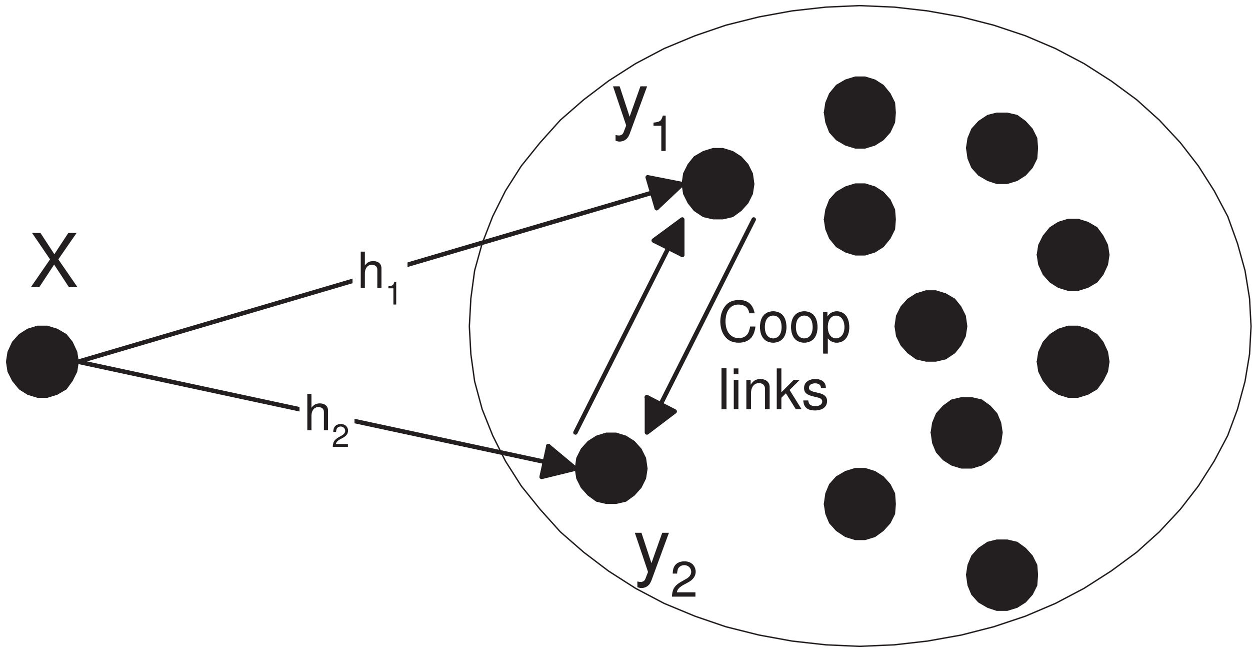
Expected Throughput vs. P_r/P_s



Two-Hop Broadcasting - Summary

- Relaying protocols for the broadcast approach within a two-hop relay fading channel were considered.
- Numerical results show that AF outperforms all other numerically tractable schemes for high SNRs, and high P_r/P_s ratios. It even approximates the broadcasting cut-set bound for $P_r/P_s \geq 10dB$.
- It was shown that for outage approach AQF is optimal in terms of relay-destination link throughput.
- **Open problems**
  - S: Continuous broadcasting, R: Outage approach.
  - Optimal DF Broadcasting, expected to outperform AF.
  - Continuous broadcasting combined with continuous SR-AQF broadcasting.

Two Colocated Cooperative Receivers



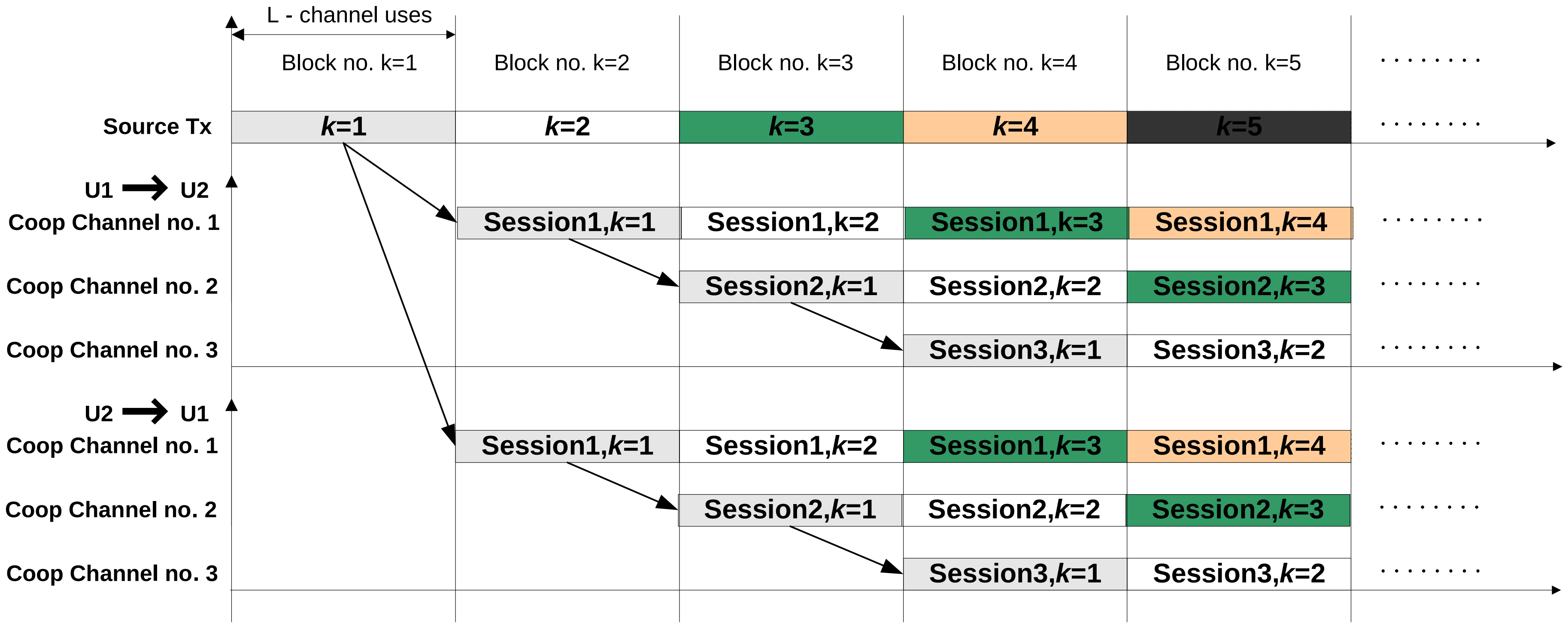
What is the optimal broadcasting strategy for communicating with a user having a colocated helper via multiple sessions?

Details in (Steiner-Sanderovich-Shamai-IT ’07).

Multi-Session Cooperation - Related Work

- Interactive decoding over discrete-memoryless-channel (DMC) channels, with results for the binary erasure channel (BEC) (Draper-Frey-Kschischang ’03).
- Both iterative and one-shot conferencing for relay channels, over Gaussian channels, with several cooperation strategies (Ng-Maric-Goldsmith-Shamai-Yates ’06).
- Rates for broadcast channel with cooperative receivers (Dabora-Servetto ’06).

Multi-Session Cooperation with $K = 3$



Cooperation Strategies:

Compress-Forward (CF)

Amplify-Forward (AF)

Cooperation channels:

Orthogonal additive white Gaussian noise (AWGN) channels, with a total power constraint for cooperation.

Multi-Session Amplify-Forward (AF)

Proposition 8 *In a multi-session AF ($K \to \infty$) cooperation strategy, the highest decodable layer is associated with an equivalent fading gain determined by*

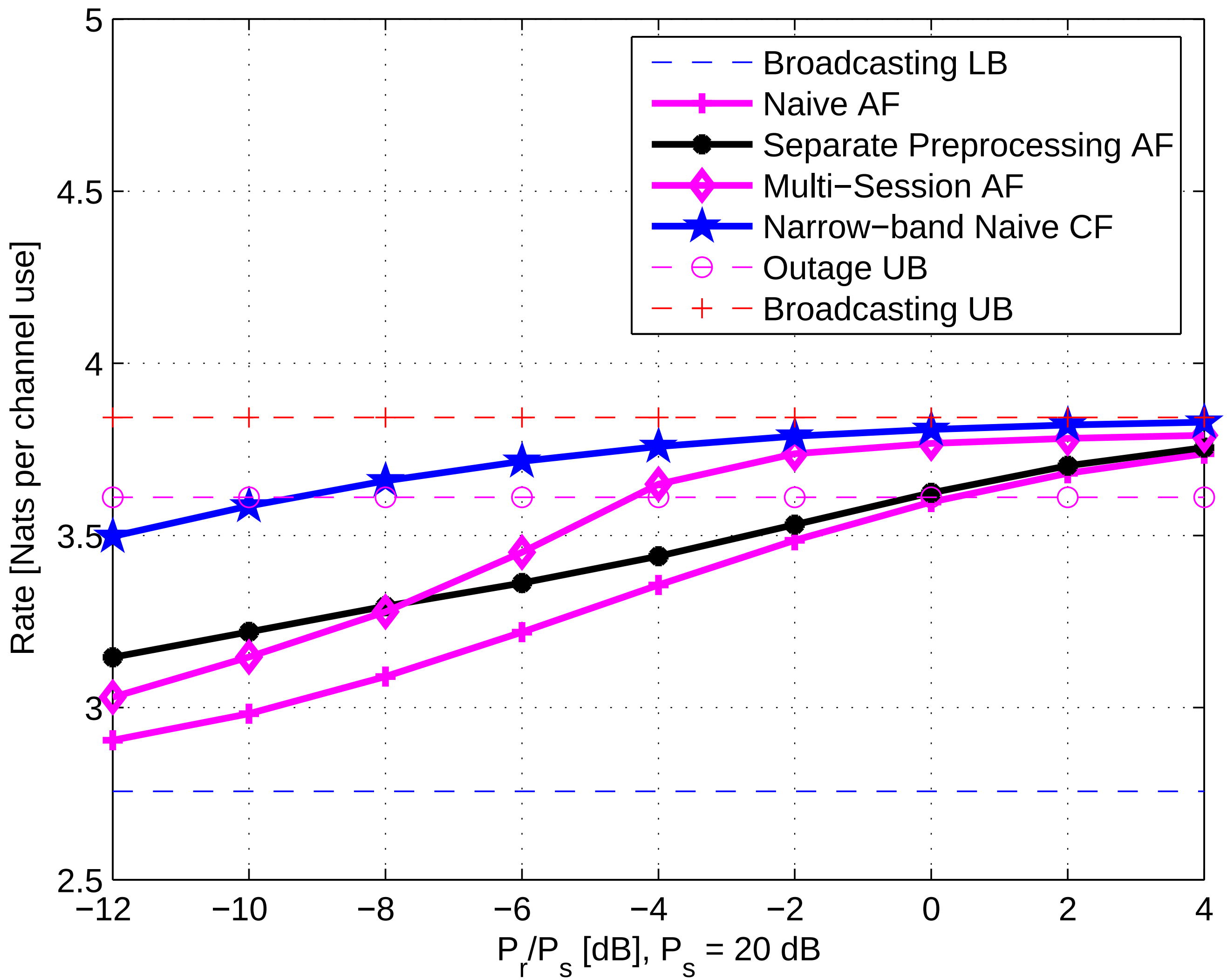
$$s_{ms} = \begin{cases} s_a^* & s_1 \geq s_2 \\ s_b^* & s_1 < s_2 \end{cases} \quad (20)$$

where s_b^* is the solution of the following equation,

$$\int_{s_2}^{s_b^*} \frac{s_1}{(s_1+s_2-\sigma)^2}[1+s_1 I(\sigma)]d\sigma = P_r, \quad s_a^* = s_1 + s_2 \frac{Z(s_b^*)}{1+Z(s_b^*)}$$

$$Z(s) = \int_{s_2}^{s} \frac{1+s_1 I(\sigma)}{(1+s_2 I(\sigma))} \frac{s_1}{(s_1+s_2-\sigma)} d\sigma$$

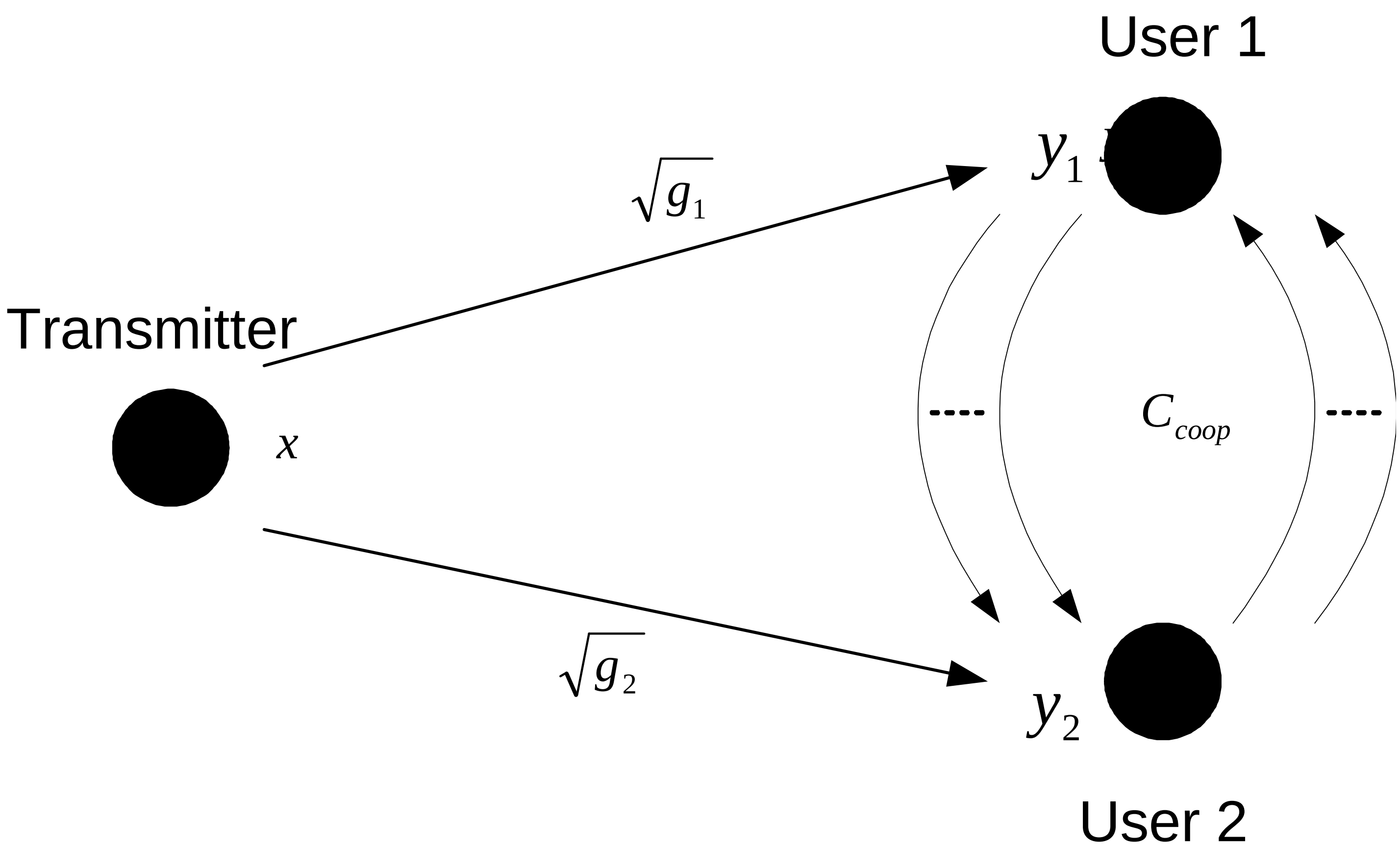
Multi-Session AF and CF



Multi-Session Broadcasting - Summary

- Combined AF and broadcast.
- Utilizing the broadcasting: single and multiple sessions.
- For CF, successive refinable Wyner-Ziv (WZ) cooperation is studied.
- No preprocessing: explicit expressions for both AF, CF.
- Multi-session AF: improvement over single session AF.
- Naive CF: very close to upper bound.
- **Open problems**
  - Optimal power distributions for $K \to \infty$.
  - Cooperation among multiple colocated users.
  - Non-orthogonal cooperation channels.

Multi-Session Cooperation over Non-Fading Channels



To Layer or not to layer?

Details in (Steiner-Sanderovich-Shamai ISIT'08).

Single-Session WZ-Compression and Binning

User 2 (g_2): WZ compression. User 1 (g_1): random binning. Total Achievable rate by both users

$$R_1 = \log\left(1 + P_s\left[g_1 + \frac{g_2}{1+D_1}\right]\right)$$

with distortion (quantization noise variance),

$$D_1 = \frac{1 + (g_1 + g_2)P_s}{(1 + g_1 P_s)(e^{R_{wz1}} - 1)}.$$

The distortion D_1 manifests itself as additional noise, on the channel $\Longrightarrow$ equivalent channel gain is $s_{eq} = g_1 + \frac{g_2}{1+D_1}$. Total cooperation rate constraint

$$C_{coop} \geq R_{wz1} + R_1 - \log(1 + g_2 P_s).$$

K-Session WZ-Compression and Binning

Proposition 9 *In a non-symmetric non-fading AWGN broadcast channel, a* ***single session (**$K=1$**) cooperation is more beneficial than a multi-session (**$K>1$**).***

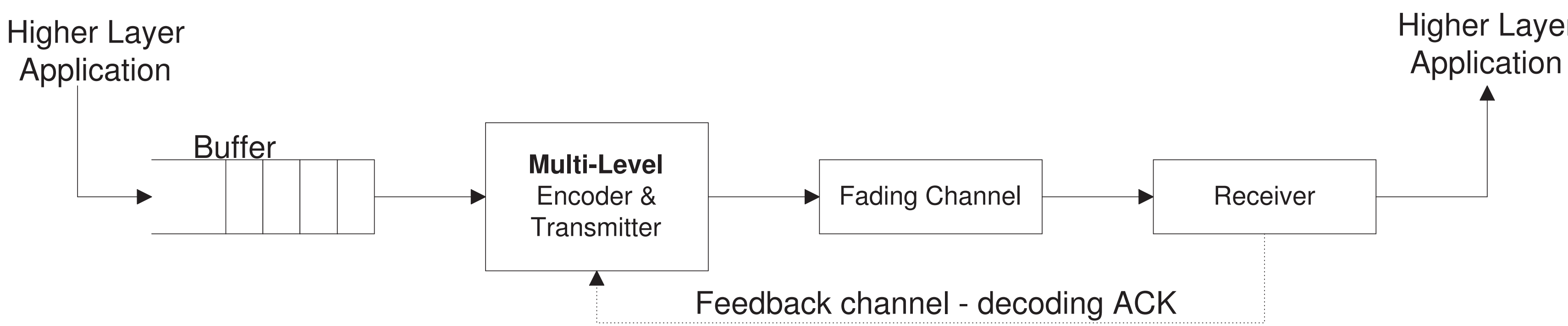
$$\max_{\substack{0 \le P_K \le P_{K-1} \le \cdots \le P_2 \le P_s, \\ 0 \le D_K \le D_{K-1} \le \cdots \le D_1, \\ \text{s.t. } C_{coop} = -\log(1+g_2 P_s) + \sum_{k=1}^{K} R_{wz,k} + R_k}} \sum_{k=1}^{K} R_k \qquad (21)$$

is maximized for $P_K = P_s$*, where* $P_k = \sum_{n=k}^{K} \Delta_n$*;* Δ_n *is the* n^{th} *layer power allocation;* $R_{wz,k}$ *refers to the successive-refinement WZ compression rate at the* k^{th} *session.*

Cooperation over Non-Fading Channels - Summary

- Single session cooperation (WZ and binning) is optimal.
- Shown that for discrete and continuous layering the special case of single layer is always better.
- Even when designing the layers such that before the first session the ’stronger’ user decodes separately the first layer, and then multi-session cooperation begins for the other layers - **single session without separate processing is optimal**.
- When considering average distortion, the results are similar (Ng-Tian-Goldsmith-Shamai ’07).
- **Open problems**
  - One of the users experiences a fading channel.
  - Two users view a fading channel, a unified layering over all uncertainties (channel and side information).

Queue and Physical Layer Model (MAC-PHY)



Is layering beneficial for cross-layer delay optimization over fading channels?

Details in (Steiner-Shamai CISS'05).

Queueing and Layering - Related Work

- Stability and delay optimal rate allocation in MAC, broadcast and relay settings (Yeh '02, '07), (Yeh-Cohen '03, '04), and for a symmetric Gaussian parallel relay network (Yeh-Berry '05).
- Queueing with adaptive modulation and coding (Liu-Zhou-Giannakis '05).
- Stability in random access networks (Shrader-Ephremides '05). Distributed queue length based algorithms for optimizing throughput (Stoylar-Liu '07).
- Delay optimization for the single server queue, with single level coding (Bettesh-Shamai '06).

Queue Model (Lindley Equation) - Zero-padding queue

$$w_{n+1} = \begin{cases} w_n + \lambda_{n+1} - R_n & w_n + \lambda_{n+1} - R_n \geq 0 \\ 0 & w_n + \lambda_{n+1} - R_n < 0 \end{cases} \quad (22)$$

n - frame index, also used as embedding point index.

λ_{n+1} - (normalized) input data to queue at a fixed rate λ.

R_n - transmission data rate random variable.

w_n - normalized data length in queue in [Nats/channel use].

Queue equation is normalized by the block length N.

Tight bounds for Queue Size

Proposition 10 *Queue average size and average delay for K-level code layering are upper and lower bounded by*

$$EW_K \geq \frac{(\Re_K - \lambda)(\sum_{i=1}^{K} p_i \Re_{K-i+1} - \lambda) - (\Re_K - \lambda)^2}{2(\sum_{i=1}^{K} p_i \Re_{K-i+1} - \lambda)} + \frac{\sum_{i=1}^{K} p_i (\Re_K - \Re_{K-i+1})^2 + \overline{p}\Re_K^2}{2(\sum_{i=1}^{K} p_i \Re_{K-i+1} - \lambda)} \quad (23)$$

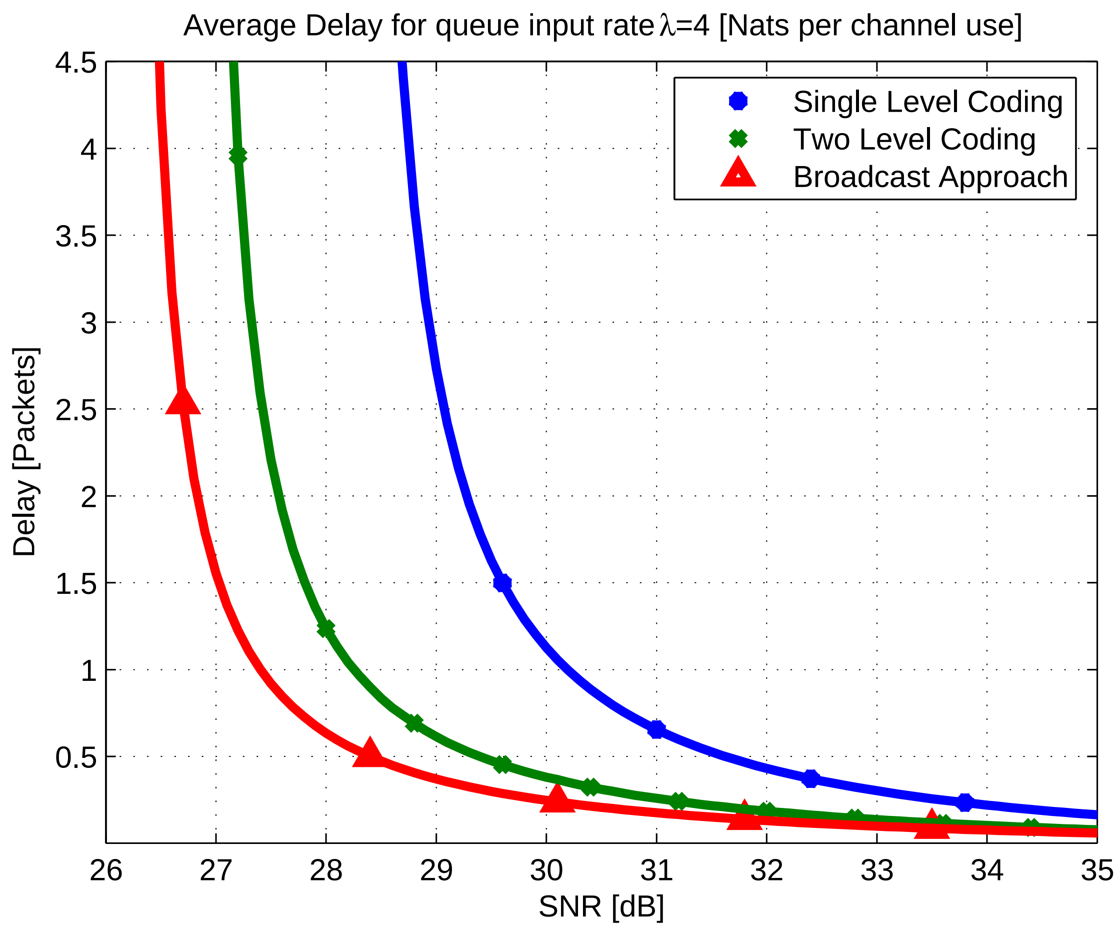
$$EW_K \leq \frac{2(\Re_K - \lambda)(\sum_{i=1}^{K} p_i \Re_{K-i+1} - \lambda) - (\Re_K - \lambda)^2}{2(\sum_{i=1}^{K} p_i \Re_{K-i+1} - \lambda)} + \frac{\sum_{i=1}^{K} p_i (\Re_K - \Re_{K-i+1})^2 + \overline{p}\Re_K^2}{2(\sum_{i=1}^{K} p_i \Re_{K-i+1} - \lambda)} \quad (24)$$

where $\Re_V \triangleq \sum_{j=1}^{V} R_j$.

Outline of Proof

1. Explicit derivation of the queue size *pdf*.
2. Laplace transform of the *pdf* $L_W(s)$.
3. The first moment $E[W]$ can be computed from the Laplace transform.
4. Since $L_Y(s)|_{s=0} = 0$ and $L_X(s)|_{s=0} = 0$, L'Hospital rule is used.
5. Common parts of the upper and lower bounds is derived.
6. Algebraic manipulation for obtaining the lower and upper bounds.

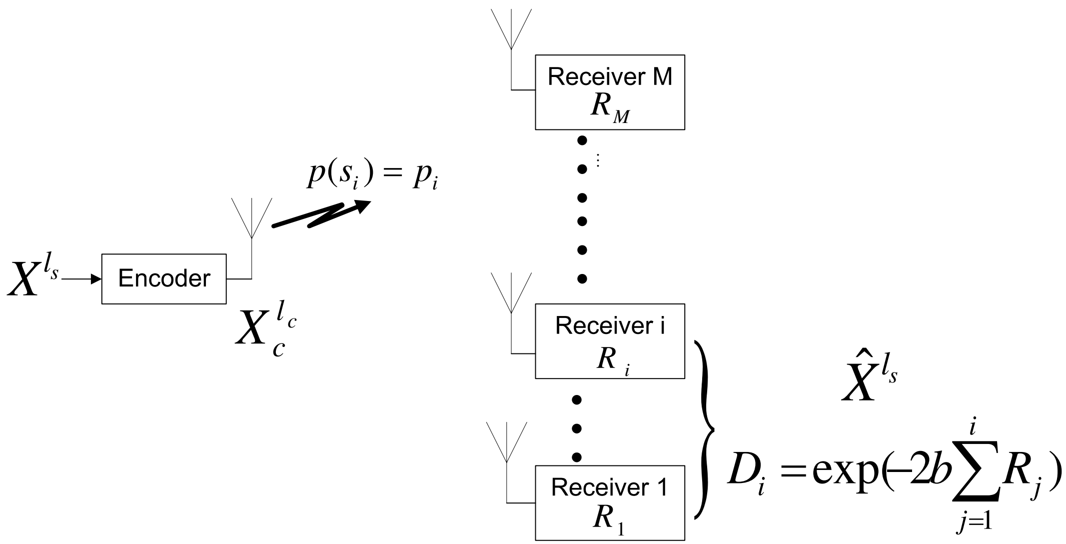
Matching Upper & Lower Delay Bounds (λ=4)



Queueing and Broadcasting - Summary

- For a layered channel encoder
  - Delay upper bound relying on the moments inequality (Daley'77) was used.
  - A tight lower bound was derived from the queue size distribution function.
  - Delay bounds for continuous layering were obtained.
- When considering ***delay*** as a performance measure, code layering may give pronounced performance gains in terms of delay, which are more definite than those associated with *throughput*.
- **Open problems**
  - Optimal power distribution for broadcasting bounds.
  - Random process for a variable queue input rate.
  - Multi-queue and broadcasting (e.g. per layer queue).

Expected Distortion



- Successive refinement (SR) - broadcast approach.
- Separate receivers designate different state values.
- The approach facilitates 'ANALOGUE' communications feature, the better the channel is the less the distortion, without any transmitter adaptation.

Expected Distortion Related Work

- Distortion exponent of some layered transmission schemes (Bhattad-Narayanan-Caire ’06).
- Recursive power allocation and minimum expected distortion in Gaussian layered broadcast coding with successive refinement (Ng-Gunduz-Goldsmith-Erkip ’07).
- Joint source channel coding for MIMO block fading channels (Gunduz-Erkip ’08).
- Hybrid digital analog joint source channel codes (Mittal-Phamdo ’02), (Caire-Narayanan ’05), (Skoglund-Phamdo-Alajaji ’06).
- Robust joint source-channel coding for delay-limited applications (Taherzadeh-Khandani ’08).

SR and Broadcast Approach

- Incremental differential rate is

$$R(s)ds \;=\; \frac{1}{2}\frac{-sI'(s)ds}{1+sI(s)}$$

- By defining $T(S) = \exp\left(2\int_0^s R(u)du\right) \Rightarrow$

$$\frac{T'(s)ds}{T(s)} \;=\; \frac{-sI'(s)ds}{1+sI(s)}$$

which is a first order differential equation solved by

$$I(s) \;=\; \frac{1}{T(s)}\int_s^\infty dr\frac{T(r)}{r^2} \;-\; \frac{1}{s}$$

Details in (Tian-Steiner-Shamai-Diggavi-IT July'08).

SR and Broadcast Approach (cont.)

- The optimization problem in variational notations is

$$\text{minimize } \overline{D} = \int_0^\infty ds\; f(s)\exp\left(-2b\int_0^s R(u)du\right) \;=\; \int_0^\infty ds\; \frac{f(s)}{T(s)^b}$$

$$\text{subject to } I(s_0) = \frac{1}{T(s_0)}\int_{s_0}^\infty dr\,\frac{T(r)}{r^2} - \frac{1}{s_0} \;=\; P$$

b - bandwidth expansion factor (ratio of channel to source codeword lengths).

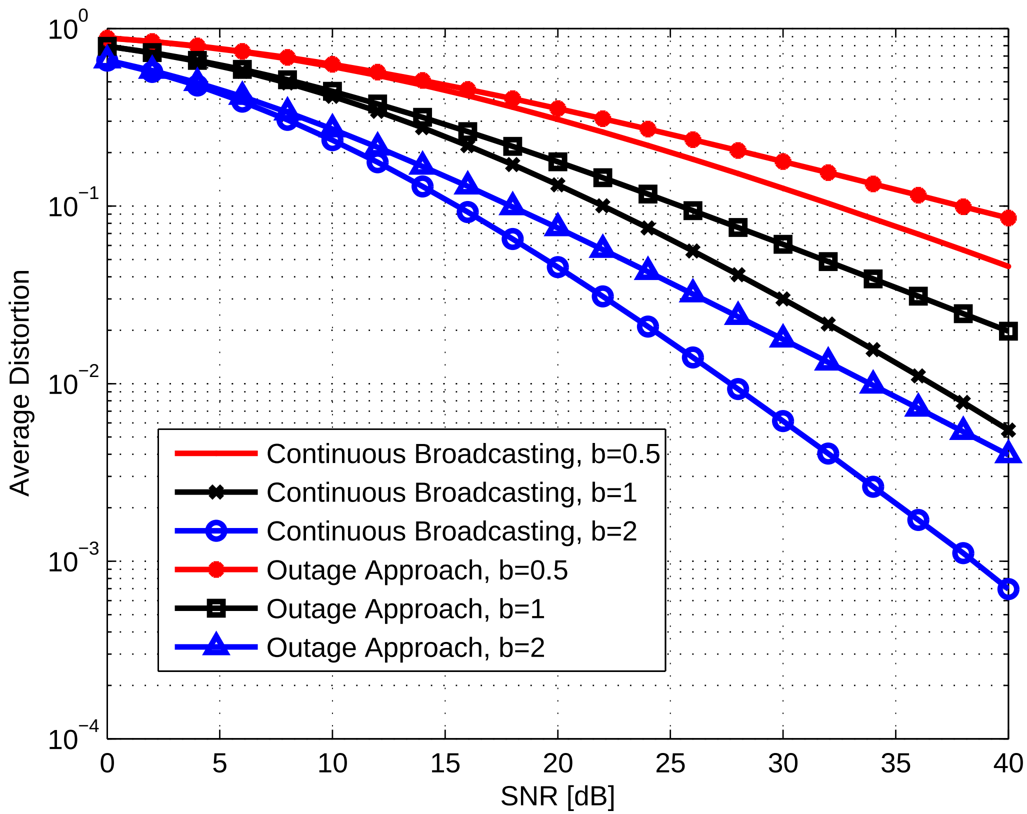
- Single interval solution for the above variational problem

$$T_{opt}(s) \;=\; \left(\frac{f(s)s^2}{f(s_0)s_0^2}\right)^{1/(b+1)} \;,\quad s \in [s_0, s_1]$$

where $I(s_0) = P,\; I(s_1) = 0$.

- Solution extends to the case of a multi-interval optimal power distribution.

Expected Distortion - Outage vs. Continuous Broadcasting



Minimal average distortion, a comparison of outage approach and broadcast approach, for different bandwidth expansion factors.

Expected Distortion - Summary

- The alternative characterization of the capacity region of the Gaussian broadcast channel (Tse '99) is used.
- A O(M) algorithm for the discrete case.
- A variational derivation of the continuous solution; analyzed for multiple positive power allocation intervals, which coincides with (Ng-Gunduz-Goldsmith-Erkip'07).
- Broadcast strategy with successive-refinement (SR) coding: source-channel separation; optimal under a common knowledge constraint (Steinberg '07).
- **Open problems**
  - Joint source/channel coding approach may improve the performance.
  - Expected distortion for non-Gaussian sources.
  - Other distortion metrics than MSE.

Research Outlook

- MAC Broadcasting (Shamai '00), (Minero-Tse '07) - equivalent fading gain of continuous layering (multi-session cooperation).
- Broadcast approach for a parallel channel (with a common message).
- The MIMO broadcast approach.
- BCC/BIR-HARQ in cooperative relay networks.
- Diversity-multiplexing tradeoff with MIMO broadcasting.
- Diversity-multiplexing-delay tradeoff of combined BCC/BIR-HARQ.
- Broadcasting with different optimization criteria: Minimum rate (Shamai '97), Multicast networks (Mirghaderi et al '08).
- Broadcasting with unreliable backhaul links (Simeone et al '09).
- Connected to new notions of Variable-to-Fixed channel coding (Verdu-Shamai '09).

Perspectives

- **Layering is a critical factor in multi-terminal comm. systems, in general (even not state dependent!)**
  - **Best achievable regions or capacities** for: MAC, Broadcast, Interference and Relay Channels/Networks.
  - Some of the recent insights:
    Interference channels: Effectiveness of the Han-Kobayashi rate splitting (layering) technique:
    (Etkin-Tse-Wang '06), (Telatar-Tse '07)
    Deterministic approaches for various networks:
    (Avestimehr-Diggavi-Tse '07), (Tse '07), (Bresler-Parekh-Tse'07).
    Broadcasting with degraded message sets: a deterministic approach (Prabhakaran-Diggavi-Tse '07).
    Deterministic MIMO broadcast model: private, common, secret messages (Ly-Liu '08).

Thank You!